Roll No. $\qquad$

## F-3855

## M.A./M.Sc. (Previous) Examination, 2022 MATHEMATICS

## Paper Fifth

(Advance Discrete Mathematics)

Time : Three Hours]
[Maximum Marks:100

Note: Attempt any two parts from each question. All questions carry equal marks.

## Unit - I

1. (a) Prepare the truth table for
$(p \Leftrightarrow q) \wedge(q \Leftrightarrow r) \Rightarrow(p \Leftrightarrow r)$
(b) If g is a homomorphism from a commutative semigroup $(S, *)$ onto a semi group $(T, \oplus)$ then show that $(T, \oplus)$ is also commutative semi
group.
(c) Show that for any commutative monoid $(M, *)$ the set of idempotent element of M forms a submonoid.
2. (a) $L=\{1,2,3,4,6,12\},(L, ' 1 ')$,
$a \vee b=L C M(a, b), a \wedge b=\operatorname{hcf}(a, b)$. Show that ( $L,{ }^{\prime} 1$ ') is a lattice but it is not complemented lattice ('1' stands for divide)
(b) Show that for Boolean algebra $\left(B, \pm i^{\prime}\right)$
$(a+b)(b+c) \cdot(c+a)=a . b+b . c+c . a$ where $a, b, c, \in B$
(c) Write the following function into conjunctive normal form $f(x, y, z)=(x+y+z),\left(x y+x^{\prime} z\right)^{\prime}$

## Unit - III

3. (a) Show that a complete graph with five vertices is not a planar graph.
(b) State and prove Euler's formula for connected planar graph.
(c) Show that a tree with $n$ vertices has $(n-1)$ edges.
4. (a) Explain the finite state machine and their transition table and transition diagrams.
(b) Minimize finite state machine $M$, where $M$ is given by the following state table.

| State | input |  | output |
| :---: | :---: | :---: | :---: |
|  | 0 | 1 |  |
| $\Rightarrow \mathrm{~S}_{0}$ | $\mathrm{~S}_{3}$ | $\mathrm{~S}_{1}$ | 1 |
| $\mathrm{~S}_{1}$ | $\mathrm{~S}_{4}$ | $\mathrm{~S}_{1}$ | 0 |
| $\mathrm{~S}_{2}$ | $\mathrm{~S}_{3}$ | $\mathrm{~S}_{0}$ | 1 |
| $\mathrm{~S}_{3}$ | $\mathrm{~S}_{2}$ | $\mathrm{~S}_{3}$ | 0 |
| $\mathrm{~S}_{4}$ | $\mathrm{~S}_{1}$ | $\mathrm{~S}_{0}$ | 1 |

(c) Find a deterministic accepter equivalent to
$M=\left(\left\{q_{0}, q_{1}, q_{2}\right\},\{a, b\}, \delta,\left\{q_{2}\right\}\right) \delta$ is given in table-

| State $/ \Sigma$ | a | b |
| :--- | :--- | :--- |
| $\rightarrow q_{0}$ | $q_{0}, q_{1}$ | $q_{2}$ |
| $q_{1}$ | $q_{0}$ | $q_{1}$ |
| $q_{2}$ | - | $q_{0}, q_{1}$ |

5. (a) Let $A=\{0,1\}$, show that the following expressions are regular expression overA.
(i) $0^{*}(0+1)^{*}$
(ii) $00^{*}(1+0)^{*}$
(iii) $(01)^{*}\left(01+1^{*}\right)$

Also find regular sets corresponding to these regular expression.
(b) Construct the grammar for the language $L=\left\{a^{n} b a^{m}|m, n, \geq 1|\right\}$ and for the string $a^{4} b a^{5}$ write the derivation.
(c) Explain regular grammar context free grammar and context sensitive grammar and give examples related to the grammars.

